The Derivative as a Function

Math 130 - Essentials of Calculus

24 February 2021

(Some) Applications of the Derivative

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- Suppose $C(q)$ is the total cost to produce q units of a good or service. Then the derivative $C'(q)$ is what is called the *marginal cost*. This basically tells us how much it costs to produce the *next* unit, which could be especially useful since the cost of producing extra units is likely to change based on just how many is desired to be produced. Likewise, we can take the derivative of a revenue function $R(q)$ to get the *marginal revenue*, which would tell us approximately how much extra income is gained by selling one extra unit.

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1. *What is the meaning of $H'(58)$? What are its units? ($58^\circ F \approx 15^\circ C$)*

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1. *What is the meaning of $H'(58)$? What are its units? ($58^\circ F \approx 15^\circ C$)*
2. *Would you expect $H'(58)$ to be positive or negative? Why?*

Now You Try It!

Example

Let $P(x)$ be the profit, in dollars, a souvenir shop makes from selling x coffee mugs during a week.

1. *Interpret the statement $P(80) = -125$.*
2. *Interpret the statement $P'(80) = 1.5$.*

The Derivative - Revisited

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For a function $f(x)$, the derivative of f is the function $f'(x)$ defined by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

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- The function $f(x)$ is said to be *differentiable* at $x = a$ if the derivative $f'(a)$ exists.

Comparing the Graphs of f and f'

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For the given function $f(x)$, (a) find $f'(x)$, (b) compare the graphs of f and f'.

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1. $f(x) = 2x + 4$
2. $f(x) = 2x^2 - 3$

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Theorem

If a function is differentiable at a number, then it is continuous there.